Study of discrete maps in terms of symbolic CTQ-analysis

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Outline

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Definition of alphabet

Denote the discrete dynamical system:

$$\mathbf{s}_{k+1} = \mathbf{g}\left(\mathbf{s}_k,\, \mathbf{p}\right), \quad \{\mathbf{s}_k\}_{k=1}^{K},$$

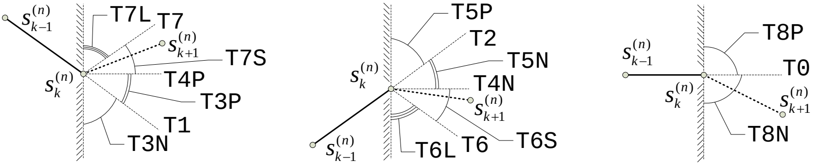
$\mathbf{s} \in \mathrm{S} \subset \mathbb{R}^N, \quad k \in \mathrm{K} \subset \mathbb{N}, \quad \mathbf{p} \in \mathrm{P} \subset \mathbb{R}^M, \quad n = \overline{1,\, N},\, k = \overline{1,\, K},\, m = \overline{1,\, M}.$

We introduce the primary mapping:

$$\left\{\mathbf{s}_{k-1}^{(n)},\, \mathbf{s}_k^{(n)},\, \mathbf{s}_{k+1}^{(n)}\right\} \Rightarrow T_k^{\alpha\varphi}|_n, \quad T_k^{\alpha\varphi} = \left[T_k^{\alpha\varphi}|_1 \ldots T_k^{\alpha\varphi}|_N\right], \quad \{T_k^{\alpha\varphi}\}_{k=1}^{K},$$

where $T^{\alpha\varphi}|_n$ – symbol of T-alphabet:

$\mathrm{T}_o^{\alpha\varphi} = \{\mathtt{T0},\, \mathtt{T1},\, \mathtt{T2},\, \mathtt{T3N},\, \mathtt{T3P},\, \mathtt{T4N},\, \mathtt{T4P},\, \mathtt{T5N},\, \mathtt{T5P},\, \mathtt{T6S},\, \mathtt{T6},\, \mathtt{T6L},\, \mathtt{T7S},\, \mathtt{T7},\, \mathtt{T7L},\, \mathtt{T8N},\, \mathtt{T8P}\}.$



Additionally define the $Q^{\alpha\varphi}|_n$ – symbol of Q-alphabet ($\mathrm{Q}_o^{\alpha\varphi} \ni Q^{\alpha\varphi}|_n$):

$$Q_k^{\alpha\varphi}|_n \equiv T_k^{\alpha\varphi}|_n \to T_{k+1}^{\alpha\varphi}|_n, \quad Q_k^{\alpha\varphi} = \left[Q_k^{\alpha\varphi}|_1 \ldots Q_k^{\alpha\varphi}|_N\right], \quad \{Q_k^{\alpha\varphi}\}_{k=1}^{K}.$$

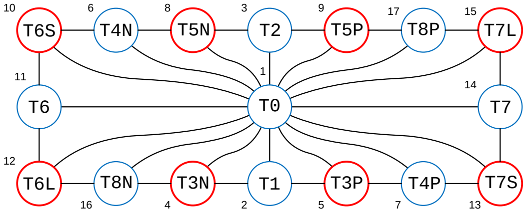
Symbolic TQ-image and Distance

Symbolic TQ-image of sequence $\{\mathbf{s}_k\}_{k=1}^{K}$

Directed graph $\Gamma^{TQ}|_n = \left\langle \mathrm{V}^{\Gamma}|_n,\, \mathrm{E}^{\Gamma}|_n \right\rangle$:

$\mathrm{V}^{\Gamma}|_n \subseteq \mathrm{T}_o^{\alpha\varphi}$ – vertex $\Gamma^{TQ}|_n$ and $\mathrm{E}^{\Gamma}|_n \subseteq \mathrm{Q}_o^{\alpha\varphi}$ – edges $\Gamma^{TQ}|_n$.

where $\mathrm{d_T}(\cdot,\cdot)$ – the shortest path between two vertices in a graph $\mathrm{D_T}$:



The primary formalisms:

- TQ-bifurcations;
- TQ-complexity;
- T-synchronization;
- Q-prediction;
- Q-control.

Main Article and Talk (in English)

- A.V.M., *Structure of Synchronized Chaos Studied by Symbolic Analysis in Velocity–Curvature Space* // Technical Physics Letters, 38:2 (2012), 155—159, arXiv: 1203.4214.
- A.V.M., *Multidimensional Dynamic Processes Studied by Symbolic Analysis in Velocity–Curvature Space* // Computational Mathematics and Mathematical Physics, 52:7 (2012), 1017—1028.
- A.V.M., *Measure of Synchronism of Multidimensional Chaotic Sequences Based on Their Symbolic Representation in a T-Alphabet* // Technical Physics Letters, 38:9 (2012), 804–808, arXiv: 1212.2724.

- A.V.M., *The possibilities by symbolic analysis in velocity-curvature space: TQ-bifurcation, symmetry, synchronization* // School-Seminar "Interaction of Mathematics and Physics: New Perspectives"/ Proceedings. – Moscow, Steklov Mathematical Institute, 2012.
- A.V.M., *Estimation complexity of chaotic oscillations in aspect of the shape of their trajectories* // XXV IUPAP Conference on Computational Physics / Book of Abstracts. – Moscow, Department of Physical Sciences of Russian Academy of Sciences, 2013, p.52.

Outline section

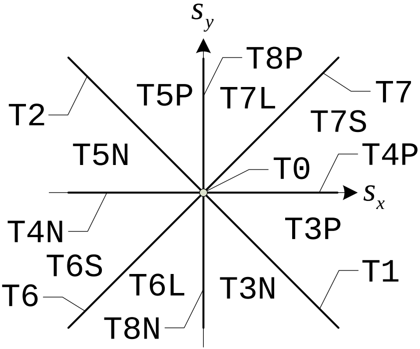
Marking T-plane

Centering the subsequence:

$$\left\{\mathbf{s}'_{k-1},\, \mathbf{s}'_k,\, \mathbf{s}'_{k+1}\right\} = \left\{\mathbf{s}_{k-1},\, \mathbf{s}_k,\, \mathbf{s}_{k+1}\right\} - \mathbf{s}_k.$$

define the coordinate system on plane:

$$F^T|_n : \left(s_x = \mathbf{s}'^{(n)}_{k-1},\, s_y = \mathbf{s}'^{(n)}_{k+1}\right).$$



$$\mathbf{s}_x = \mathbf{s} - \mathbf{g}(\mathbf{s},\, \mathbf{p}),$$

$$\mathbf{s}_y = \mathbf{g}^2(\mathbf{s},\, \mathbf{p}) - \mathbf{g}(\mathbf{s},\, \mathbf{p}),$$

where: $\mathbf{g}^w(\mathbf{s},\, \mathbf{p}) = \mathbf{g}(\mathbf{g}(\ldots \mathbf{g}(\mathbf{s},\, \mathbf{p}),\, \mathbf{p}),\, \mathbf{p})$.

Finally:

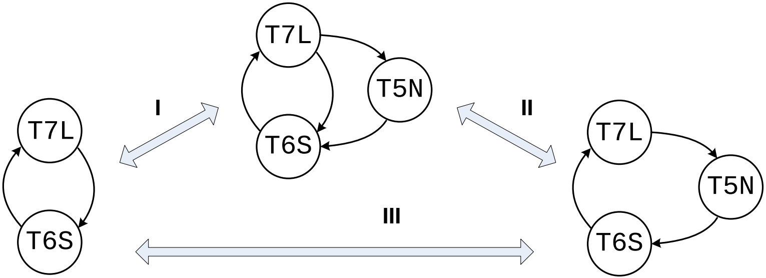
$$\mathbf{M}^T(\mathbf{s},\, \mathbf{p}) = [\mathbf{s}_x,\, \mathbf{s}_y,]\,.$$

Map TQ-bifurcation

Definition

TQ-bifurcation in a discrete dynamical system is:

$$ {}^{a}\Gamma^{TQ} \xrightarrow[\mathbf{p}=\mathbf{p}_b]{\text{TQ-bif}} {}^{b}\Gamma^{TQ}, \quad {}^{a}\Gamma^{TQ} \neq {}^{b}\Gamma^{TQ}. $$



Outline section

Is there any difference on chaos?

Logistic Map:

$$s_{k+1} = 4\,\lambda\, s_k(1 - s_k), \quad s \in (0,\,1), \quad \lambda \in (0,\,1],$$

$$\lambda > \lambda_\infty = 0.892486418\ldots \text{ – chaos zone}$$

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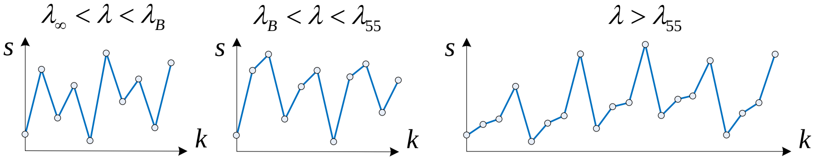

The qualitative difference of chaotic trajectories.

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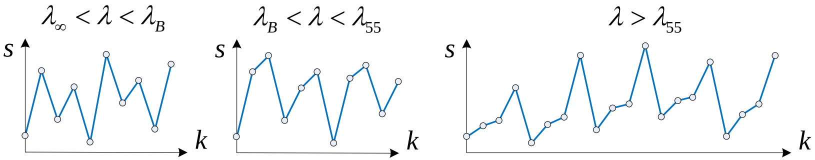

The qualitative difference of chaotic trajectories.

Thank you for your attention!