Generalized synchronization of multidimensional chaotic systems in terms of symbolic CTQ-analysis

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Motivation

Generalized Synchronization and Symbolic Dynamics

- Generalized Synchronization of chaotic oscillations in form

\[ y = F[x, \tau] \]

is very important phenomena in physics (and not only in physics).

- But many important problems in this field remain unsolved: reliable detection, time structure, etc.

- In their turn Symbolic Dynamics is a very strongly substantiated tool for the analysis of nonlinear dynamical systems.

- It allows one to investigate complicated phenomena in systems such as chaos, strange attractors, hyperbolicity, structural stability, controllability, etc.

We have combined positions and obtain the new tool:

Generalized T-Synchronization.
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Definition of alphabet

Denote the discrete dynamical system:

\[ s_{k+1} = f(s_k, p), \quad \{s_k\}_{k=-\infty}^{\infty}, \]

\[ s \in S \subset \mathbb{R}^N, \quad k \in K \subset \mathbb{N}, \quad p \in P \subset \mathbb{R}^M, \quad n = 1, N, k = 1, K, m = 1, M. \]

We introduce the primary mapping:

\[ \{s_{k-1}^{(n)}, s_k^{(n)}, s_{k+1}^{(n)}\} \Rightarrow T_{\alpha \varphi} |_n, \quad T_{\alpha \varphi} = [T_{\alpha \varphi}^{(1)} \cdots T_{\alpha \varphi}^{(N)}], \quad \{T_{\alpha \varphi}^{(r)}\}_{k=1}^{K}, \]

where \( T_{\alpha \varphi}^{(r)} |_n \) – symbol of T-alphabet:

\[ T_{\alpha \varphi} = \{T0, T1, T2, T3N, T3P, T4N, T4P, T5N, T5P, T6S, T6, T6L, T7S, T7, T7L, T8N, T8P\}. \]

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The general idea of T-synchronization

Remark

Let sequence $\{s_k\}_{k=1}^K$ of dimension $N$ is formed by the combination of the phase variables of $N$ one-dimensional dynamical systems; i.e. $s_k^{(n)}$ is the value of the phase variable of the $n$th system.

Definition

We will assume that the dynamical systems are synchronous at time instant $k$th in the sense of T-synchronization if the condition $J_k = 1$ is satisfied, where

$$J_k = \begin{cases} 1 & T_k^{\alpha \varphi} |_1 = \ldots = T_k^{\alpha \varphi} |_n = \ldots = T_k^{\alpha \varphi} |_N, \\ 0 & \text{otherwise}. \end{cases}$$
Basic measures of T-synchronization

Anti-synchronization $s^{(n)}_k \rightarrow -1 \cdot s^{(n)}_k$ – inversion of symbols $T^\alpha \varphi |_n$:

- $T0 \leftrightarrow T0$,
- $T1 \leftrightarrow T2$, $T3N \leftrightarrow T5P$, $T3P \leftrightarrow T5N$, $T4N \leftrightarrow T4P$,
- $T6S \leftrightarrow T7S$, $T6 \leftrightarrow T7$, $T6L \leftrightarrow T7L$, $T8N \leftrightarrow T8P$.

Lag-synchronization – shift between components:

$$\{ T^\alpha \varphi |_1 \rightarrow T^\alpha \varphi |_{1+h_1}, \ldots, T^\alpha \varphi |_{N} \rightarrow T^\alpha \varphi |_{N+h_N} \}.$$ 

Partial integral coefficient of synchronism:

$$\delta^s_{m,h} = \frac{1}{K^* + 1 - k^*} \sum_{k=k^*}^{K^*} J_k \{ m, h \},$$

where $k^* = 1 + \max (h_1, \ldots, h_N)$ and $K^* = K + \min (h_1, \ldots, h_N)$.

Total integral coefficient of synchronism:

$$\delta^s = \max_m \max_h \delta^s_{m,h}, \quad 0 \leq \delta^s \leq 1.$$
We introduced the concept of a domains of two types:

*synchronous domain* $SD$

$$SD_r : \{ J_{k'} = 1, J_{k''} = 0 \lor k'' = 0, J_{k'''} = 0 \lor k''' = K + 1 \} ,$$

$$k' = b^SD_r, b^SD_r + L_r, \quad k'' = b^SD_r - 1, \quad k''' = b^SD_r + L^SD_r + 1,$$

*desynchronous domain* $\overline{SD}$

$$\overline{SD}_r : \{ J_{k'} = 0, J_{k''} = 1 \lor k'' = 0, J_{k'''} = 1 \lor k''' = K + 1 \} ,$$

$$k' = b^SD_r, b^SD_r + L^SD_r, \quad k'' = b^SD_r - 1, \quad k''' = b^SD_r + L^SD_r + 1,$$

$\lor$ is the symbol of the logical operation OR.
The time structure of T-synchronization

The spectral density function of synchronous domains SD:

\[ H^{SD}[L] = \sum_{r=1}^{R^{SD}} \delta[L_{r}^{SD}, L], \]

where \( \delta[\circ, \circ] \) is the Kronecker delta and \( L = 1, K \).

The entropy of the structure of synchronous domains SD:

\[ E^{SD} = -\sum_{i=1}^{K} P^{SD}[i] \ln P^{SD}[i], \quad P^{SD}[L] = \frac{H^{SD}[L]}{\sum_{i=1}^{K} H^{SD}[i]}. \]

The general idea of Generalized T-synchronization

**Definition**

We will assume that the dynamical systems are synchronous at time instant $k$th in the sense of generalized T-synchronization if the condition $J_k = 1$ is satisfied, where

$$J_k = \begin{cases} 1 & T_k^{\alpha \varphi} \in M_{snc}^{FT}, \\ 0 & \text{otherwise}. \end{cases}$$

where $M_{snc}^{FT}$ is set of symbols $T^{\alpha \varphi}$, which define synchronize state.

The symbols $T^{\alpha \varphi}$ are encoded in form of $T_i^1 \cdots T_i^n \cdots i_N$.

Basic requirements for the set $M_{snc}^{FT}$:

- $|M_{snc}^{FT}| \leq |T_\circ^{\alpha \varphi}|$,

- $|M_{snc}^{FT}|i_n \leq 1, \forall T_i \in T_\circ^{\alpha \varphi}, n = 1, N$,

where $|\circ|$ is cardinality of set.
Construction of the set $M_{sn}^{FT}$

The objective function to fill the set $M_{sn}^{FT}$:

- Integral coefficient of synchronism: $\frac{1}{K} \sum_{i=1}^{K} i H^{SD}[i] \equiv \delta^s \to \max$,

- Length of synchronous domain: $\{\max L^{SD} : H^{SD}[L^{SD}] \geq 1\} \to \max$,

- ...

The number of variants is filling of the set $M_{sn}^{FT}$:

- Complete synchronization: $N_{sn}^{FT} = 1$,

- Antisynchronization: $N_{sn}^{FT} = 2^{N-1}$,

- Generalized synchronization:

$$N_{sn}^{FT} = \prod_{n=0}^{T-1} (T - n)^{N-1} = \left( \frac{2P(3, T-1)}{T+1} \right)^{N-1},$$

where $P$ is Pochhammer symbol, $T = |T_o^{\alpha \varphi}|$. 
The number of variants is filling of the set $M_{snc}^{FT}$.

Samples ($T = 17$ is standard T-alphabet):

- $N = 2, \quad N_{snc}^{FT} = 355\,687\,428\,096\,000$,
- $N = 3, \quad N_{snc}^{FT} = 126\,513\,546\,505\,547\,170\,185\,216\,000\,000$,
- $N \gg 2, \quad$ Curse of dimensionality!

Who is to blame? What to do?

It is an open problem!

Variant: breadth-first search with a cut-off of bad branches.
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The object of analysis is the time series of exchange rates of some world currencies (US dollar [USD], Euro [EUR], Japanese Yen [JPH], Swiss Franc [CHF], and British Pound [GBP] against Russian ruble).

The analyzed period is from 01.01.1999 to 31.12.2014.

The original data are taken from the official web-site of the Central Bank of Russia (Bank of Russia, exchange rates, www.cbr.ru/eng/).
Synchronization of USD and EUR

Short sample: USD and EUR.

- Complete synchronization: \( \delta^s = 0.174492 \),

- Antisynchronization: \( \delta^s = 0.219433 \),

- Generalized synchronization: \( \delta^s = 0.222948 \).

Short conclusions:

- The structure of synchronicity USD and EUR is more complex than the Complete or Anti-.

- Generalized synch is a combination of Anti- and Complete- synch.
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In this report, we proposed an original approach to the evaluation and analysis of generalized synchronization of chaotic sequences.

The real experiment demonstrated the efficiency measures of generalized T-synchronization.

The developed tools expand methods of computational physics for study various phenomena in nonlinear multi-dimensional dynamical systems.

At the moment, we are resolving one open problem:

- The effective algorithms for filling set $M^{FT}_{snc}$.

Thank you for your attention!